Pion mass effects on axion emission from neutron stars through NN bremsstrahlung processes

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Abstract

The rates of axion emission by nucleon-nucleon bremsstrahlung are calculated with the inclusion of the full momentum contribution from a nuclear one pion exchange (OPE) potential. The contributions of the neutron-neutron (nn), proton-proton (pp) and neutron-proton (np) processes in both the non-degenerate and degenerate limits are explicitly given. We find that the finite-momentum corrections to the emissivities are quantitatively significant for the non-degenerate regime and temperature-dependent, and should affect the

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existing axion mass bounds. The trend of these nuclear effects is to diminish the emissivities.

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1. Introduction

The search for new particles/interactions beyond the Standard Model is one of the most important tasks of particle physics. While several candidates and proposals may be considered as "exotic", in the sense of not being required by the data, it is generally agreed that there are some possibilities definitely expected as minimal extensions of it. Axions ([1]-[2]) belong to the latter category, as expected from the Peccei-Quinn ([3]-[5]) solution to the strong CP-problem. Axions are pseudo Nambu-Goldstone boson associated with the spontaneous breaking of the Peccei-Quinn symmetry. Their masses and couplings are directly related to this symmetry-breaking scale. Viable versions of axionic models include the KSVZ ([1],[6]) and DFSZ ([7]-[8]) axions coupled to hadrons only and to leptons and hadrons respectively.

In addition to experimental efforts for a direct detection of axions, astrophysical and cosmological arguments have played a key role in their search. Actually, stringent bounds have been obtained from the consideration of horizontal branch stars ([9]-[11]), white dwarf cooling and SN1987A neutrino pulse duration ([12]), among others. A general review of these arguments has been given in [13] (see also [14]-[16] for a thorough account).

One of the main ingredients for an accurate calculation of axion mass bounds is the emissivity in the nucleon bremsstrahlung reaction $NN \rightarrow NNa$, thought to be dominant in important astrophysical events, such as newly born neutron stars. Calculations and discussions on the applicability of the emissivity formulae were given in [8],[12]-[19]. Quite generally, in these papers, the calculations were performed for a one-pion exchange free nucleon gas, leaving aside important issues later incorporated and assessed, such as the effects of correlations between nucleons (see for instance [20]). Attempts to link the emissivity to laboratory data have been also made. We would like to present in this communication a reassessment of the bremsstrahlung emissivity including the full momentum dependence of the matrix elements. We found that the hitherto neglected dependence produces large temperature-dependent corrections to the rates independently of the many body effects, a

feature that points by itself to a revision of some of the astrophysical bounds.

Axion emission are important for the evolution of stars particularly for (hot) neutron stars (NS). For the conditions relevant to the core of hot NS just after their formation ($T \sim 30-60$ MeV, $\rho \geq \rho_0$ (with $\rho_0 \equiv 2.7 \times 10^{14} gcm^{-3}$ the nuclear matter density) the dominant emission process are the nucleon-nucleon (NN) bremsstrahlung ([19])

$$n + n \rightarrow n + n + a \qquad (a)$$

$$p + p \rightarrow p + p + a \qquad (b)$$

$$n + p \rightarrow n + p + a \qquad (c)$$

Previous calculations of the axion emissivities have been performed by Iwamoto ([9],[17]) in the degenerate (D) limit. For the NN interaction he used a OPE potential in the Born approximation and found the expressions of the energy-loss rates for all the processes above.

Later on, Brinkmann and Turner [19] calculated the axion emission rates in the nondegenerate (ND) limit and for a general degeneracy, for all three processes (a -c). They also could check the result of Iwamoto for the equal-nucleon cases in the (D) limit. For the NN interaction they used, however, constant nuclear matrix elements. The same results for the ND limit was also obtained previously by Turner in [18]. They reached the conclusion that ND regime is a better approximation of the axion emissivities for the conditions characteristic for a newly born neutron star. Other calculations considering also a OPE potential where performed by Raffelt and Seckel [20]. They studied the axion emission rates of $NN \rightarrow NNa$ processes in order to determine their $S_A(\omega)$ structure function for NN interactions in neutron stars. With their calculations they concluded that the inclusion of pion mass effects do not reduce the axion emissivities by more than 50% even for ND regime. Our results will show that the contributions of pion mass to the coresponding emissivities due to nuclear effects are temperature dependent, and for a certain temperature interval are larger than this, as we will see later. Therefore, in this article we pay special attention to the ND regime but we present also the results for D regime. For the contribution of the np process we consider different chemical potentials for neutrons and protons, and as a result we can span different degeneracy degrees for the two species.

Analogously to the case of neutrino pair emission ([21]-[26]), one of the main difficulties for the calculation of axion emissivities is the appropriate

treatment of the strong NN interaction. In Iwamoto's calculations for the D limit the effects were included by replacing the nucleon momenta by their Fermi values in the angular part of the phase-space integrals.

In the present work we include the full dependence on nuclear momenta of the nuclear matrix elements (NME) in the calculations of the axion emission rates by the NN bremsstrahlung processes (a - c). Our results for ϵ_{aNN} separate explicitly the part corresponding to constant NME, which corresponds to the high-momentum limit of the previous works ([9],[12]-[22]), from the part including the missing nuclear effects due to the nucleon momenta dependence of the NME to facilitate the comparisons and further applications. We compare our results with those obtained by Brinkmann and Turner ([19]) for the ND limit, while for the degenerate regime we compare with those obtained in Ref. ([9]) and ([19]).

2. Calculations

The axion emission rate by NN bremsstrahlung is given by Fermi's Golden Rule formula (see for instance [19])

$$\epsilon_{aNN} = (2\pi)^4 \int \left[\Pi_1^4 \frac{d^3\mathbf{p}_i}{(2\pi)^3 2E_i} \right] \frac{d^3\mathbf{p}_a}{(2\pi)^3 2E_a} E_a \left(S \times \Sigma |M|^2 \right) \delta^4(P) F(f) \quad (1)$$

where $F(f) = f_1 f_2 (1 - f_3)(1 - f_4)$ is the product of Fermi-Dirac distribution functions of the initial (1,2) and final (3,4) nucleons, $f_i = \left(\exp^{\frac{E_i - \mu_i}{T}} + 1 \right)^{-1}$. In Eq. (1) $\mathbf{p}_i$ and E_i (i=1,4) are the nucleon momenta and energies, while $\mathbf{p}_a$ and E_a are the corresponding axion quantities; S is a symmetry factor taking into account the identity of the particles (1/4 for nn and pp channels and 1 for the np channel) and μ_i are the chemical potentials of the nucleons.

In the non-relativistic limit $E_i \sim m + \frac{\mathbf{p}_i^2}{2m}$. Using the non-dimensional quantities [19] $y = \hat{\mu}/T$ ($\hat{\mu} = \mu - m$) and $u_i = \mathbf{p}_i^2/2mT$, the expressions of the Fermi-Dirac functions read $f_i = (\exp^{u_i - y_i} + 1)^{-1}$. The degenerate (D) limit satisfies $y >> 1$, while in the non-degenerate (ND) limit $y << -1$. For $S \times \Sigma |M|^2$ we use the following expressions for the nuclear matrix elements (in the OPEP approximation):

$$S \times \Sigma |M|^2 = S \times \frac{256}{3} \cdot g_{ai}^2 m^2 \left(\frac{f}{m_\pi} \right)^4 \cdot M_{NN} \quad (2)$$

where

$$g_{ai} = Cm_N/(f_a/N)$$

C is a dimensionless factor of order unity, which is model dependent,

$$m_n \simeq m_p = 940MeV = m_N \rightarrow g_{an} \simeq g_{ap} = g_{ai} = C \cdot 5.64 \cdot 10^{-10}$$

($f_a = 10^{10}GeV$ is the Peccei-Quinn symmetry breaking scale, $N = 6$ represents the color anomaly of the Peccei-Quinn symmetry).
For the nn and pp the momentum-dependent factors M_{NN} read

$$M_{nn} = \left(\frac{|\mathbf{k}|^2}{|\mathbf{k}|^2 + m_\pi^2}\right)^2 + \left(\frac{|\mathbf{l}|^2}{|\mathbf{l}|^2 + m_\pi^2}\right)^2 + \frac{(1-\beta)|\mathbf{k}|^2 \cdot |\mathbf{l}|^2}{(|\mathbf{k}|^2 + m_\pi^2)(|\mathbf{l}|^2 + m_\pi^2)} \quad (3)$$

with $\beta = 3\langle(\underline{\mathbf{k}} \cdot \underline{\mathbf{l}})^2\rangle$ ($\underline{\mathbf{k}}, \underline{\mathbf{l}}$ being the coresponding unit vectors for $\mathbf{k}$ and $\mathbf{l}$), while for the np process

$$M_{np} = \left(\frac{|\mathbf{k}|^2}{|\mathbf{k}|^2 + m_\pi^2}\right)^2 + 4\left(\frac{|\mathbf{l}|^2}{|\mathbf{l}|^2 + m_\pi^2}\right)^2 + 2(1-\beta)\frac{|\mathbf{k}|^2 \cdot |\mathbf{l}|^2}{(|\mathbf{k}|^2 + m_\pi^2)(|\mathbf{l}|^2 + m_\pi^2)} \quad (4)$$

where $\mathbf{k} = \mathbf{p}_1 - \mathbf{p}_3$ and $\mathbf{l} = \mathbf{p}_1 - \mathbf{p}_4$ are the nucleon direct and exchange transfer momenta, respectively. The last (exchange) terms in the above expressions arise from interference of two different reaction amplitudes. They contain contributions from the scalar product $(\mathbf{k} \cdot \mathbf{l})^2$, which have been estimated ([12],[19]) by replacing them by their average values (denoted by β) in the phase-space. There are two numerical values for β in the literature: $\beta = 1.0845$, in Ref. [19] and $\beta = 1.3078$ in Ref. [20], but this difference (explained by Raffelt and Seckel) produces changes in our final results of only 1%. Since we compare our results especially with those of Ref. [19], we use that value for β. Thus, from kinematical constraints β =0 in the D regime, while it is 1.0845 in the ND regime (see the expression above). We have used for the np process, the NME of [19], with equal coupling constants for protons and neutrons.

We follow the procedure of Brinkmann and Turner [19] to derive the ND limit, by performing the transformation to the center-of-mass system

$$\mathbf{p}_+ = \frac{\mathbf{p}_1 + \mathbf{p}_2}{2}; \quad \mathbf{p}_- = \frac{\mathbf{p}_1 - \mathbf{p}_2}{2}; \quad \mathbf{p}_{3c} = \mathbf{p}_3 - \mathbf{p}_+; \quad \mathbf{p}_{4c} = \mathbf{p}_4 - \mathbf{p}_+$$

$$\Rightarrow \mathbf{p}_1 = \mathbf{p}_+ + \mathbf{p}_-; \quad \mathbf{p}_2 = \mathbf{p}_+ - \mathbf{p}_-; \quad \mathbf{p}_3 = \mathbf{p}_+ + \mathbf{p}_{3c}; \quad \mathbf{p}_4 = \mathbf{p}_+ + \mathbf{p}_{4c} \qquad (5)$$

From these relations and the conservation of momentum (axion momentum is neglected) we find $\mathbf{p}_{4c} = -\mathbf{p}_{3c}$. We define also the dimensionless quantities

$$u_i = \frac{\mathbf{p}_i^2}{2mT}(i = \overline{1,4}); \quad u_+ = \frac{\mathbf{p}_+^2}{2mT}; \quad u_- = \frac{\mathbf{p}_-^2}{2mT}; \quad u_{3c} = \frac{\mathbf{p}_{3c}^2}{2mT}, \qquad (6)$$

$$cos\gamma_1 = \frac{\mathbf{p}_+\mathbf{p}_-}{|\mathbf{p}_+||\mathbf{p}_-|}; \quad cos\gamma_c = \frac{\mathbf{p}_+\mathbf{p}_{3c}}{|\mathbf{p}_+||\mathbf{p}_{3c}|}; \quad cos\gamma = \frac{\mathbf{p}_-\mathbf{p}_{3c}}{|\mathbf{p}_-||\mathbf{p}_{3c}|}; \qquad (7)$$

From the definition of the u variables above, and the conservation of energy, one can easily deduce the following relations

$$u_{1,2} = u_+ + u_- \pm 2(u_+u_-)^{1/2}cos\gamma_1; \quad u_{3,4} = u_+ + u_{3c} \pm 2(u_+u_{3c})^{1/2}cos\gamma_c;$$

$$u_- = u_{3c} + E_a/2T \qquad (8)$$

Let us now address the OPE potential. Following the method used in our previous papers ([23],[24]), and after some lengthy algebra, one can express the matrix element M_{nn} (eq.(4)) in terms of the scalar combinations $|\mathbf{k}|^2 + |\mathbf{l}|^2$ and $|\mathbf{k}|^2 \cdot |\mathbf{l}|^2$. Finally we expressed these NME in the following compact form

$$S \times \Sigma|M|^2 = \frac{64m^2g_{ai}^2}{3}\left(\frac{f}{m_\pi}\right)^4 \left[(3-\beta) - |M_{nn}|^2_{nucl}\right] \qquad (9)$$

where

$$|M_{nn}|^2_{nucl} = m_\pi^2 \frac{A_{nn} - B_{nn} \cdot C_\phi^2}{C - D \cdot C_\phi^2 - E \cdot C_\phi^4} \qquad (10)$$

The coefficients A_{nn}, B_{nn}, C, D and E of eq.(10) are polynomials depending on the parameters m, T and m_π and of variables u_- and u_{3c} (for their full expressions, see Appendix A). Also we used the notation $C_\phi = cos\gamma_1 cos\gamma_c + sin\gamma_1 sin\gamma_c cos\phi$, with ϕ the angle between the vectors $\mathbf{p}_+$ and $\mathbf{p}_-$.

Thus, the contribution of NME is split into a constant term,obtained also by Brinkmann and Turner ([19]), Raffelt and Seckel([20])) - which represents just its high-momentum limit (i.e. the limit to which the expression (3) converges when the pion mass is neglected compared to the nucleon momentum transfer) and a reduction term (see Appendix A) to be evaluated. After an

approximation which is numerically accurate within 1%, we succeeded to integrate the expression of $|M_{nn}|^2$ over the angles and finally we could express the axion emission rate in the ND limit in the following form

$$\epsilon_{aNN}^{ND} = \epsilon_{aNN}^{ND}(0)\left(1 - \frac{I_{nucl}^{ND}(NN)}{(3-\beta)I_0^{ND}}\right) \tag{11}$$

where

$$\epsilon_{aNN}^{ND}(0) = 2.68 \times 10^{-4} g_{ai}^2 e^{2y} m^{2.5} T^{6.5} (f/m_\pi)^4 \tag{12}$$

is the expression calculated by Brinkmann and Turner [19] and I_0^{ND} and $I_{nucl}^{ND}(NN)$ are double integrals over u_- and u_{3c}

$$I_0^{ND} = \int_0^\infty \int_0^{u_-} \sqrt{(u_- u_{3c})}(u_- - u_{3c})^2 e^{-2u_-} du_- du_{3c} \tag{13}$$

$$I_{nucl}^{ND}(NN) = \frac{\pi m_\pi^2}{mT} \int_0^\infty \int_0^{u_-} \sqrt{(u_- u_{3c})}(u_- - u_{3c})^2 e^{-2u_-} \times \left(\frac{(7-\beta)m_1 + 4(3-\beta)(u_- + u_{3c})}{(2u_- + 2u_{3c} + m_1)^2}\right) du_- du_{3c} \tag{14}$$

with $N = n, p$, $m_1 = m_\pi^2/mT$.

A similar procedure for the np process yields

$$S \times \Sigma |M|^2 = \frac{256 m^2 g_{aN}^2}{3}\left(\frac{f}{m_\pi}\right)^4 \left[(7-2\beta) - |M_{np}|^2_{nucl}\right] \tag{15}$$

where

$$|M_{np}|^2_{nucl} = m_\pi^2 \frac{A_{np} - C_{np}\cdot C_\phi^2}{C - D\cdot C_\phi^2 - E\cdot C_\phi^4} + C_\phi \frac{B_{np} - D_{np}\cdot C_\phi^2}{C - D\cdot C_\phi^2 - E\cdot C_\phi^4} \tag{16}$$

and $g_{aN} = [(7-2\beta)/3]g_{ai}^2$ is the effective axion nucleon coupling for the np case (see expression A.1 in ref.[19]),

and final expressions analogous to Eq.(11) and (12):

$$\epsilon_{anp}^{ND} = \epsilon_{anp}^{ND}(0)\left(1 - \frac{I_{nucl}^{ND}(np)}{(7-2\beta)I_0^{ND}}\right) \tag{17}$$

$$\epsilon_{anp}^{ND}(0) = 2.68 \times 10^{-4} g_{aN}^2 e^{y_1+y_2} m^{2.5} T^{6.5} (f/m_\pi)^4 \tag{18}$$

The correction integral of the third term $I_{nucl}^{ND}(NN)$ is replaced by

$$I_{nucl}^{ND}(np) = \frac{\pi m_\pi^2}{mT} \int_0^\infty \int_0^{u_-} \sqrt{(u_- u_{3c})}(u_- - u_{3c})^2 e^{-2u_-} \times$$

$$\left(\frac{4(7-2\beta)(u_- + u_{3c}) + (17-2\beta)m_1}{(2u_- + 2u_{3c} + m_1)^2} \right) du_- du_{3c} \tag{19}$$

Using the same procedure, we calculated the emissivities for the $nn(pp)$ processes in the D limit. In this case, in performing the integrals over the momenta and energies, we used the method of integration adopted in Refs.[9],[17] and [25]. The expressions of the emissivities are

$$\epsilon_{aNN}^D = \epsilon_{aNN}^D(0) \left(1 - \frac{I_{nucl}^D(NN)}{3I_0^D} \right) \tag{20}$$

where

$$\epsilon_{aNN}^D(0) = \left(\frac{31}{3780\pi} \right) \left(\frac{g_{ai}^2}{\hbar^5 c^7} \right) \left(\frac{f}{m_\pi} \right)^4 m_n^2 p_F(N)(kT)^6$$

or

$$\epsilon_{aNN}^D(0) = \frac{31\sqrt{2}}{3780\pi} m^{2.5} T^{6.5} m_\pi^{-4} g_{ai}^2 f^4 y^{1/2} \tag{21}$$

(in natural units and $p_F(N) = 2mTy$)
with N $= n$ or p, and

$$I_0^D = \int_0^\infty z^3 \left(\frac{4\pi^2 + z^2}{e^z - 1} \right) dxdz \tag{22}$$

$$I_{nucl}^D(NN) = \frac{\pi m_\pi^2}{mT} \int_0^\infty \int_0^{x_f} z^3 \left(\frac{4\pi^2 + z^2}{e^z - 1} \right) \times \left(\frac{6(2x+z) + 7m_1}{(2x+z+m_1)^2} \right) dxdz \tag{23}$$

Similar calculations for the np process yields a final expression analogous to Eq. (17):

$$\epsilon_{anp}^D = \epsilon_{anp}^D(0) \left(1 - \frac{I_{nucl}^D(np)}{7I_0^D} \right) \tag{24}$$

with

$$\epsilon_{anp}^D(0) = \frac{31\sqrt{2}}{3780\pi} m^{2.5} T^{6.5} m_\pi^{-4} g_{aN}^2 f^4 y_m^{1/2} (1 - \Delta y / 2y_m) \tag{25}$$

(expression obtained in ref. [19]), where $y_m = (y_1 + y_2)/2$, $\Delta y = |y_1 - y_2|/2$, and the correction integral

$$I^D_{nucl}(np) = \frac{\pi m_\pi^2}{mT} \int_0^\infty \int_0^{x_f} z^3 \left(\frac{4\pi^2 + z^2}{e^z - 1} \right) \times$$

$$\times \left(\frac{14(2x+z) + 17m_1}{(2x+z+m_1)^2} \right) dxdz \qquad (26)$$

With these expressions at hand, we discuss the results in next section.

3. Results

In Figs. 1 and 2 we plotted the dependence on temperature of the relative corrections of the emissivities, $\Delta\epsilon/\epsilon_0 = (\epsilon_0 - \epsilon)/\epsilon_0$ (where ϵ_0 is the emissivity calculated in the high momentum limit - see Ref. [19], while ϵ is the emissivity determined with our method), for ND regime (Fig. 1 - *nn/pp* processes and Fig.2 - *np* process) due to pion mass effects (see Eqs. (11), (17)). One observes that for this regime, the relative corrections to the previous results for the emissivities are quite important and temperature dependent. For all processes, the corrected emissivities are reduced with 30% to 85%, depending on temperature. Figs. 3 and 4 show the same relative emissivities, this time for the D regime (Fig. 3 - *nn/pp* processes and Fig.4 - *np* process). In this case, the corrected emissivities are reduced at most with 11%. So, for this regime the contributions from pion mass effects (see Eqs. (20),(24)) do not seriously affect the corresponding emission rates (a result qualitatively reported in [20]).The dependence of these corrections on temperature is smoother in this case than for the ND regime. For more confidence, we give in Table 1 the absolute values for $\Delta\epsilon/\epsilon_0$ for ND and D regimes. We remark a very similar behaviour of the *nn/pp* and *np* relative corrections, for both regimes.

$T[MeV]$	$(\Delta\epsilon/\epsilon_0)_{ND}^{nn/pp}$	$(\Delta\epsilon/\epsilon_0)_{ND}^{np}$	$T[MeV]$	$(\Delta\epsilon/\epsilon_0)_{D}^{nn/pp}$	$(\Delta\epsilon/\epsilon_0)_{D}^{np}$
25	0.8503	0.8490	1	0.1030	0.1070
30	0.7189	0.7180	2	0.0960	0.0997
35	0.6234	0.6227	3	0.0900	0.0931
40	0.5503	0.5497	4	0.0846	0.0874
45	0.4923	0.4918	5	0.0800	0.0824
50	0.4455	0.4450	6	0.0756	0.0780
55	0.4070	0.4066	7	0.0720	0.0740
60	0.3738	0.3735	8	0.0686	0.0705
65	0.3459	0.3455	9	0.0653	0.0673
70	0.3224	0.3220	10	0.0627	0.0643
75	0.3010	0.3008	20	0.0440	0.0435

Table 1: Relative correction to the emissivities (relative emissivities) for ND and D regime, for all NN bremsstrahlung processes (nn, pp, np). Here $\Delta\epsilon/\epsilon_0 = (\epsilon_0 - \epsilon)/\epsilon_0$, with ϵ - the emission rates determined with our method and ϵ_0 - the emissivities previously obtained in Ref. [19]

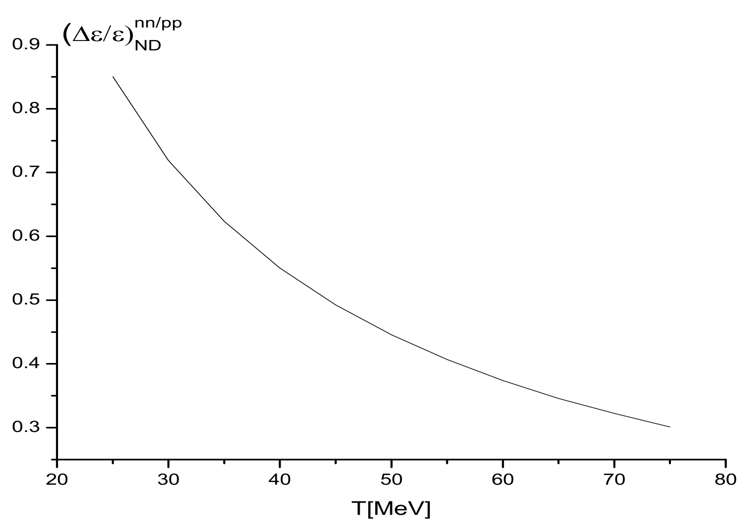

Figure 1: Relative correction to the emissivities (Relative emissivities) due to pion mass effects for the ND regime, nn, pp processes.

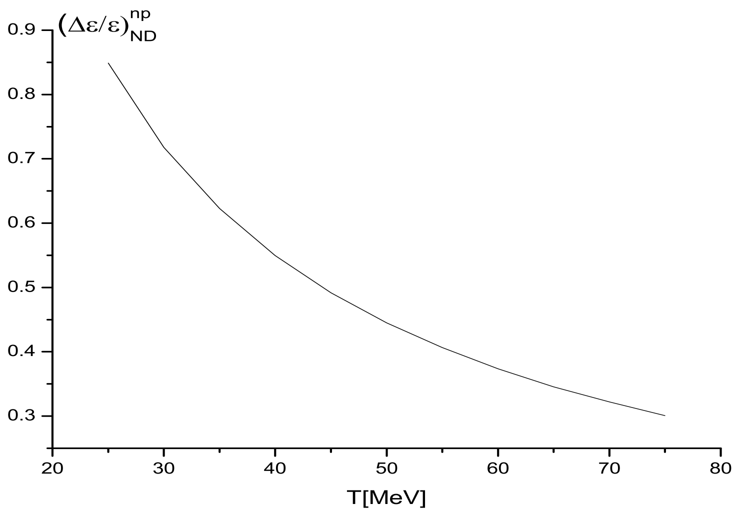

Figure 2: Relative emissivities due to pion mass effects for the ND regime, np process.

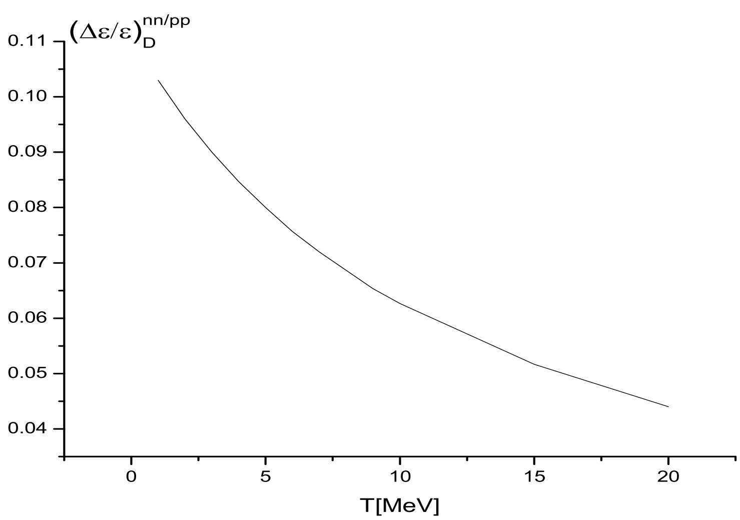

Figure 3: Relative correction to the emissivities due to pion mass effects for the D regime, nn, pp processes.

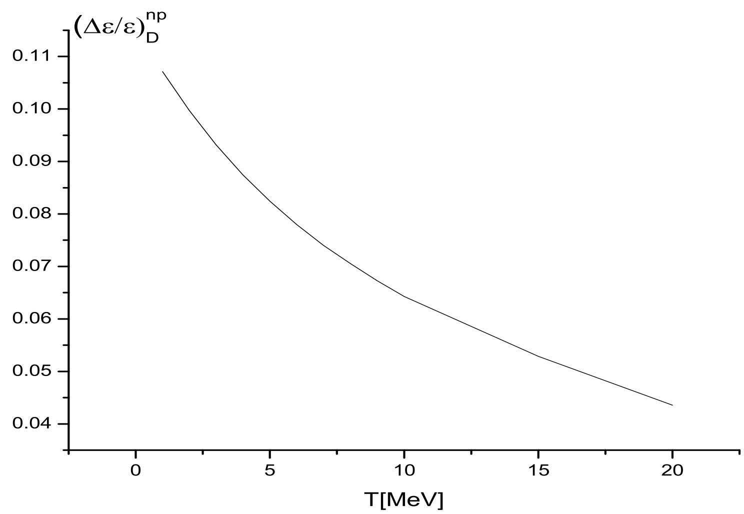

Figure 4: Relative emissivities due to pion mass effects for the ND regime, np process.

4. Conclusions

In this paper we presented a method of determination of the NN axion-bremsstrahlung emissivities, for all processes, in both ND and D regimes, based on the inclusion of the full dependence of the NME on nuclear momenta.

Starting with the ND case, worked out for the nn, pp and np cases, we found substantial reductions to the emissivities due to the combined effects of momentum-dependence in the temperature-dependent integrals - Eqs. (14),(19). While these effects were somewhat foreseen in previous works ([19]), here we present an explicit calculation and quantitative results. We claim, based on the present results, that all axion mass limits that employed ϵ_{aNN} should be revised (the ones used in numerical simulation supernova codes), especially those for which the ND limit is highly relevant, such as SN1987A neutrinos ([12]). As it stands, the suppression of ϵ_{aNN} can be important and this feature postpones one to obtain a firm bound on m_a for both popular KSVZ ([1],[6]) and DSVZ ([7],[8]) axion models.

For the D case, our method allowed to check that the leading terms (Eq. (21), (25)) coincide with Iwamoto ([9],[17]) and Brinkmann and Turner ([19]) results (see for example Eqs.(2.6) and (2.8) of Ref. [17]), and also that the

reductions are up to 11% for all three processes (a-c).

We also mention that other nuclear effects like spin-density fluctuations ([27],[28]) or short range effects (TPEP) can further reduce the coresponding axion emissivities.We are currently calculating the NN axion-bremsstrahlung emissivities by including two pion exchange effects (through a TPEP) that can be mimicked by the exchange of a ρ meson ([29]). These effects are important at distances below 2 fm ([30]). Preliminary results show us a further reduction of the axion emission rates compared with the case presented here. The calculations will be reported in a future paper. For the moment it is fair to point out that apparently minor sources of error become actually important for the problem, already at the OPE approximation level, as explicitly shown above. Also, it is worth to mention that our method might be used to improve calculations for other physical processes of neutrino and axion emission in NS, in which strong interactions are also involved.

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A. Appendix A

We present here the correction matrix elements for all the cases and processes, which have not been presented in section 2 (Calculations). Starting with ND regime, for nn and pp processes, the correction matrix element is

$$|M_{nn}|^2_{nucl} = m_\pi^2 \frac{A_{nn} - B_{nn} \cdot C_\phi^2}{C - D \cdot C_\phi^2 - E \cdot C_\phi^4} \qquad (A.1)$$

where

$$A_{nn} = 2(3-\beta)m_t^3 U_+^3 + 5(3-\beta)m_\pi^2 m_t^2 U_+^2 + 4(3-\beta)m_\pi^4 m_t U_+ \qquad (A.2)$$

$$B_{nn} = 4m_t^2 U_p[2(3-\beta)m_t U_+ + (7-\beta)m_\pi^2] \qquad (A.3)$$

$$C = m_t^4 U_+^4 + 4m_t^3 m_\pi^2 U_+^3 + 6m_\pi^4 m_t^2 U_+^2 + 4m_\pi^6 m_t U_+ + m_\pi^8 \qquad (A.4)$$

$$D = 8m_t^2 U_p (m_t U_+ + m_\pi^2)^2 \qquad (A.5)$$

$$E = 16 m_t^4 U_p \qquad (A.6)$$

with

$$m_t = 2mT; U_+ = u_- + u_{3c}; U_p = u_- u_{3c} \tag{A.7}$$

In ND case, for the np process we obtained the correction matrix elements and the corresponding coefficients (C, D and E are the same) as follows:

$$|M_{np}|^2_{nucl} = m_\pi^2 \frac{A_{np} - C_{np} \cdot C_\phi^2}{C - D \cdot C_\phi^2 - E \cdot C_\phi^4} + \frac{B_{np} C_\phi - D_{np} \cdot C_\phi^3}{C - D \cdot C_\phi^2 - E \cdot C_\phi^4} \tag{A.8}$$

where

$$A_{np} = (7 - 2\beta)[2m_t^3 U_+^3 + 5m_\pi^2 m_t^2 U_+^2 + m_\pi^4 (m_\pi^2 + 4m_t U_+)] \tag{A.9}$$

$$B_{np} = 12 m_t^2 U_+ U_p^{1/2} (m_t U_+ + m_\pi^2) \tag{A.10}$$

$$C_{np} = 4[2(7 - 2\beta) m_t^3 U_+ U_p + (17 - 2\beta) m_t^2 m_\pi^2 U_p] \tag{A.11}$$

$$D_{np} = 48 m_t^3 U_p^{3/2} \tag{A.12}$$

For the D regime, the expressions for the correction matrix elements and for the corresponding coefficients are obtained by taking $\beta = 0$ in previous relations (A.1-A.12).

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